CURRICULUM
OF
MATHEMATICS
BS (4–YEAR)

2008

HIGHER EDUCATION COMMISSION
ISLAMABAD
CURRICULUM DIVISION, HEC

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PREFACE

3
Curriculum development is a highly organized and systematic process and involves a number of procedures. Many of these procedures include incorporating the results from international research studies and reforms made in other countries. These studies and reforms are then related to the particular subject and the position in Pakistan so that the proposed curriculum may have its roots in the socio-economics setup in which it is to be introduced. Hence, unlike a machine, it is not possible to accept any curriculum in its entirety. It has to be studied thoroughly and all aspects are to be critically examined before any component is recommended for adoption.

In exercise of the powers conferred by sub-section (1) of section 3 of the Federal Supervision of Curricula Textbooks and Maintenance of Standards of Education Act 1976, the Federal Government vide notification No. D773/76-JEA (cur.), dated December 4th, 1976, appointed the University Grants Commission as the competent authority to look after the curriculum revision work beyond class XII at the bachelor level and onwards to all degrees, certificates and diplomas awarded by degree colleges, universities and other institutions of higher education.

In pursuance of the above decisions and directives, the Higher Education Commission (HEC) is continually performing curriculum revision in collaboration with universities. According to the decision of the special meeting of Vice-Chancellor’s Committee, the curriculum of a subject must be reviewed after every 3 years.

A committee of experts comprising of conveners from the National Curriculum Revision of HEC in Basic, Applied Social Sciences and Engineering disciplines met in April 2007 and developed a unified template to standardize degree programs in the country to bring the national curriculum at par with international standards, and to fulfill the needs of the local industries. It also aimed to give a basic, broad based knowledge to the students to ensure the quality of education. The new BS degree shall be of 4 years duration, and will require the completion of 130-136 credit hours. For those social sciences and basic sciences degrees, 63.50% of the curriculum will consist of discipline specific courses, and 36.50% will consist of compulsory courses and general courses offered through other departments.

For the purpose of curriculum revision various committees are constituted at the national level, comprising of senior teachers nominated by universities, degree awarding institutions, R&D organizations and respective accreditation councils. The National Curriculum Revision Committee for Mathematics in a meeting held on February 15-16, 2008 at the HEC Regional Center, Lahore in continuation of its earlier meeting held on October 26-27, at HEC Islamabad revised the curriculum in light
of the unified template. The final draft prepared by the National Curriculum Revision Special Committee, duly approved by the competent authority, is being circulated for implementation in the concerned institutions.

DR. RIAZ-UL-HAQ TARIQ
Member Academics

June 2008
Abbreviations Used:
CRC. Curriculum Revision Committee
VCC. Vice-Chancellor’s Committee
EXP. Experts
COL. Colleges
UNI. Universities
PREP. Preparation
REC. Recommendations
Fig. 1. Philosophy, contents and pedagogical techniques — their relationship with curriculum development
MINUTES OF THE NATIONAL CURRICULUM REVISION COMMITTEE FOR MATHEMATICS

A final meeting of National Curriculum Revision Committee (NCRC) for Mathematics was held at HEC Regional Center, Lahore on February 15 & 16, 2008, which was a follow-up of preliminary meeting held at C&T Building, HEC, H-8, Islamabad on October 26 & 27, 2007. The following experts attended the meetings:

1. **Professor Dr. Syed Arif Kamal**  
    Convener  
    Professor  
    Department of Mathematics  
    University of Karachi  
    Karachi

2. **Professor Dr. Ahsanullah Baloch**  
    Member  
    Professor  
    Department of Basic Sciences and Related Studies  
    Mehran University of Engineering and Technology  
    Jamshoro, Sindh

3. **Professor Dr. Barbu Berceanu**  
    Member  
    Professor  
    The Abdul Salam School of Mathematical Sciences  
    (SMS), Lahore

4. **Professor Dr. Anwar Chaudhry**  
    Member  
    Director  
    Center for Advanced Studies  
    In Pure and Applied Mathematics (CASPAM)  
    Bahauddin Zakariya University  
    Multan

5. **Professor Dr. Lal Chand**  
    Member  
    Professor and Chairman  
    Department of Mathematics  
    Shah Abdul Latif University  
    Khairpur

6. **Professor Dr. Karamat Hussain**  
    Member  
    Professor and Chairman  
    Department of Mathematics  
    Government College University  
    Lahore.

7. **Professor Dr. Faqir Muhammad**  
    Member
On October 26, 2007, the preliminary meeting started with recitation of verses from the Holy Quran by Professor Dr. Ahsanullah Baloch. Member, HEC, Professor Dr. Riaz-ul-Haq Tariq, welcomed the participants and briefed them about the curricula-development exercise taken by HEC. He, also, requested the participants to develop 4-year BS program according to the standardized template agreed upon during the meeting of conveners of NCRC on April 30, 2007 as well as review MS and PhD programs. During this short talk Professor Tariq expressed the desire to develop communication and problem-solving skills in the students of mathematics.

The Committee, unanimously, appointed Professor Dr. Syed Arif Kamal as convener and Professor Dr Shahid S. Siddiqi as secretary. Professor Kamal, in his opening remarks, thanked the participants for working tirelessly with him to develop BS, MS and PhD schemes of studies. The following items were discussed:

**Item No. 1:** Look into the need and the necessity of compulsory courses, Mathematics I, Mathematics II and Introduction to Computers for mathematics majors.

**Resolution No. 1:** It was resolved that Introduction to Computers is needed for mathematics majors. The contents of Mathematics I and
Mathematics II should be different for students majoring in mathematics because they have a much intensive program of studies. The students of mathematics should study computational mathematics aspects in the above courses. Mathematics I should focus on programming languages. The students should know one language in depth. Mathematics II should focus on computing-software applications. The students should learn one package in detail. The committee, also, proposed course contents for Mathematics I and Mathematics II to be taken by students from disciplines other than mathematics.

**Item No. 2:** Out of 4 electives, how many should be in the field of specialization and how many outside the field of specialization?

**Resolution No. 2:** It was resolved that 3 electives should be in the field of specialization and 1 outside the field of specialization.

**Item No. 3:** Prioritize the Core Courses into the following categories:

- a) Must know (Discipline-Specific-Foundation Courses)
- b) Should know (Major Courses)
- c) Nice to know (Electives)

**Resolution No. 3:** The courses were classified into the following categories:

- a) **Foundation Courses:**
  - Algebra I, Algebra II, Algebra III, Calculus I, Calculus II, Calculus III, Complex Analysis, Ordinary-Differential Equations, Real Analysis I, Real Analysis II

- b) **Major Courses:**

- c) **Electives:**
  - Listed separately near the end of BS scheme.

**Item No. 4:** Look into the need and the necessity of 3-4 courses of Physics for mathematics majors (as physics majors are required to study 4 courses of mathematics in some universities).

**Resolution No. 4:** It was resolved that physics may be helpful for those, who opt for general courses in the area of physical sciences, but not for
other areas, e. g., natural sciences, management sciences, and social sciences

**Item No. 5:** Establish the depth and the breadth in terms of flow chart for the BS scheme:

- **a**) Precedence Graphs (Depth: Pre-requisites)
- **b**) Influence Graphs (Breadth: Co-requisites)

**Resolution No. 5:** A subcommittee headed by Professor Dr. Syed Arif Kamal was designated the task to prepare the flow chart. The flow chart was discussed in the meeting and approved.

**Item No. 6:** Learning outcomes at the end of each year of BS programs of study.

**Resolution No. 6:** A subcommittee headed by Professor Dr. Barbu Berceanu was given the task to prepare the learning outcomes. The prepared outcomes were discussed in the meeting and approved.

The convener compiled the schemes of studies, in particular setting the BS scheme according to the agreed-upon template, in the light of discussion of the committee. The schemes were uploaded on the homepage of convener.

On February 15, 2008, the final-special meeting started with recitation from the Holy Quran by Professor Baloch. Member HEC, Professor Tariq, in his opening remarks suggested that the BS scheme might include 2 social-science courses, instead of one. He, further, proposed that the compulsory-mathematics courses might be customized for different disciplines, e. g., mathematics for biological scientists, mathematics for chemists, mathematics for linguists, mathematics for physicists and mathematics for social scientists. A similar suggestion was made in a paper by convener [Kamal SA and Siddiqui KA, *Physics Education (India)*, April-June 1989, pp. 53-61]. The participants, critically, scrutinized and finalized the BS scheme of studies, which was uploaded on website of convener. The draft was, also, e-mailed to all participants. Suggestions received were incorporated in the final version.

NCRC expressed the feelings that much more than mere curriculum development is needed to bring mathematics teaching at par with the developing countries, like Korea, Malaysia, Singapore, and others:

**Teachers’ Training**
To handle such a demanding curriculum teachers need to be trained. Teachers should be prepared to teach all core courses. This can be achieved through refresher and special courses for college and university teachers. The teachers should maintain office hours to help students overcome their difficulties, guide and counsel students for further studies and career.

The fundamental responsibility of a mathematics teacher is to first enhance the teaching of basic theory, to execute closely such education of scientific methodology as induction, contrast, analogy and imitation.

**Teaching Load**
Teaching load should be balanced so that a teacher can find time to do research. The class size should not exceed 25 students. Each senior instructor should be provided at least two junior instructors (teaching assistants), one to teach tutorial sessions and the other to grade problems. In courses having laboratory, an additional teaching assistant is needed to conduct the laboratory.

**Textbooks and Teaching Aids**
To teach effectively there is a need to make available textbooks at cheaper prices. These textbooks should be supplemented with solution manuals for instructors and study guides for students (these are not the “keys”, or the “guides” available in the market) as well as softwares, slides, films, charts, demonstrations, models and journals. Students should be encouraged to make drawings, models and softwares, which illustrate difficult concepts.

**Getting the Best Students**
In Pakistan, the brightest students compete for places in engineering and medical colleges as well as computer science and information technology programs. Those unable to find places in the professional institutions come to Mathematics Departments. In order to attract the very best to mathematics one needs to make effective contact with the prospective students by organizing open houses, summer programs for high-school seniors, lectures in schools and colleges, etc. Mathematics should be presented as fun through games and simulations, dramatic demonstrations, activities done outdoors and illustrations of mathematics in sports. Usefulness of mathematics can be demonstrated by daily-life applications and as preparation of a scientific career. The goal is to enable the students to see the bridge leading them from mathematics to technology passing through physics and engineering.

**Students’ Feedback**
To promote effective learning and teaching there should be continuous feedback from students — much more than the confidential-evaluation form filled by each student at the end of a semester. End-of-term-confidential evaluation shall help students of the following batch, not the batch studying at that time.

**Regulating Body**

There is need for a regulating body, which could maintain quality control over the choice of textbooks and journals published within the country. Such a body may be ‘Pakistan Mathematics Society’, which could publish one or a series of journals, e.g., Pakistan Journal of Mathematics, instead of every university bringing out its own journal.
THE BACHELORS PROGRAM IN MATHEMATICS

Degree Awarded: Bachelor of Studies (BS): Four-Year Program

Rationale: In the present-day-civilized society, numeracy (knowledge of basic mathematics) is the single most important skill needed for survival next to communication skills.

Objectives: The BS scheme of studies aims to establish the base for lifelong education by creating essential concepts and equipping the student with necessary techniques, needed to start a career of research, development, teaching or financial applications involving mathematics. The equations of mathematics are made to speak through physics, which is the formulation of general laws applying, mainly, inductive logic. Engineering is modeling from the general laws to create practical systems. Technology is the implementation and the adaptation of a laboratory model to create a working system, which could be mass-produced. The journey from mathematics to technology could be considered as a journey from the abstract to the concrete, mathematics being in books, in the minds of philosophers, physics making contact with outside world, technology becoming the stage, where one enjoys the blessings. In other disciplines, like, economics, life sciences, a similar route is followed to transform the abstract formulae and equations developed by a pure mathematician to live and vibrating applications in everyday world. The training of mathematicians should inculcate creative thinking and make them capable to critically analyze a problem, taking them from various stages of concept building to equip them with problem-solving skills. Problem solving in the classroom should prepare them to problem solving in the laboratories, and, eventually, problem solving in the industry. After completing the BS program, the students should:
be able to experience mathematics.

b) appreciate importance of mathematics and visualize its applications in every walk of life.

c) understand and use the basic structures applicable in modern sciences.

**Entrance Requirements:**

HSC (Pre-Engineering Group) or equivalent

**Duration of the Program:**

4 years (8 semesters, each semester comprising of 18 weeks); students NOT from the Pre-Engineering Group have to attend intensive courses in Mathematics, Physics and Chemistry during the semester breaks of first and second years depending on their deficiencies. Students cannot get the degree of BS before completing 4 years of study.

**Total Credit Hours:**

133 (maximum 18 hours per semester); 1 credit hour (theory) means 1 contact hour per week throughout the semester; 1 credit hour (laboratory) means 3 contact hours of practical work.

**Compulsory Courses:**

9 (25 credit hours)

**General Courses:**

8 (24 credit hours); 6 to be chosen from physical sciences (one course must be physics), natural sciences, social sciences or management sciences (18 credit hours); one course “Introduction to Psychology” (3 credit hours) and another “Introduction to Sociology” (3 credit hours)

**Core Courses (Prioritization):**

These are prioritized into the following categories:

a) Must know (Discipline-Specific-Foundation Courses) 10 (28 credit hours)

b) Should know (Major Courses) 12 (44 credit hours, including guest/students' seminars)

c) Nice to know (Electives)
4 (12 credit hours);

**Electives A:** 3 (9 credit hours) [Electives in Specialization]

**Electives B:** 1 (3 credit hours); mathematics courses outside the field of specialization [Free Electives]

**Project:** 3 credit hours, with accompanying written report and presentation (may be taken in lieu of fee elective)

**Core Courses (Depth & Breadth):** The depth and the breadth is illustrated in terms of flow chart for the BS scheme (Figure 2)

a) Precedence Graphs (Depth: Pre-requisites)

b) Influence Graphs (Breadth: Co-requisites)

**Academic Standards:** In order to bring the standard of education at par with the developed countries, the notion that anyone who gets admitted to a university shall end up with a degree should be abolished.

a) Those, who fail in more than two courses in a semester, are, automatically, dismissed.

b) Those, who fail in one or two courses, may retake the examination during the 40-day grace period.

c) Those, who have not cleared all courses after the grace period are dismissed.

d) There shall be no choice in quizzes, hourlies and final examinations. All questions given have to be attempted.

**Class Size:** 50 (maximum) for lectures; 25 (maximum) for tutorial and laboratory session; course supervisor assisted by one or more associate instructors

**Minimum Lectures:** Before final examination could be conducted minimum number of lectures to be delivered must be at least 13 per credit hour (i.e., 26 for 2-hour course; 39 for 3-hour course; 52 for 4-hour course)

**Attendance** Students are required to maintain 75% attendance in order to sit in the final examination.
**Requirement:**
Dean, under special circumstances, may condone attendance between 60-74%. Attendance registers, bearing the list of students, should be provided to the instructors on the first day of classes. Instructors take the attendance at the start of every class. At the end of each class associate instructor keys in the hard-copy attendance record to a centralized database. On the first of every month list of students having attendance less than 75% is displayed on the notice board. Those having attendance less than 60% should be required to attend fresh classes.

**Summer Activity:**
Students could be assigned independent study. Third- and fourth-year students may be placed in local industry and financial institutions for internships during semester breaks. During the fourth year the students should receive career counseling for suitable placement after completion of BS degree.

**Seminars:**
During each semester student must participate in Guest Seminar (presentations by eminent mathematicians on theoretical aspects and applications) and Students’ Seminar (presentations by students); Guest Seminars and Students’ Seminars are held fortnightly during the same time slot (6 credit hours) — graded on the basis of presentation and write-up by the student, a weekly quiz on the contents of Guest/Students’ Seminar held during the previous week.

**Comprehensive Viva:**
Conducted by senior faculty members of the department at the end of eighth semester.

**Qualifications for Course Supervisors (Instructors):**
Faculty holding PhD or MS degree is entitled to teach lecture session of a course.

**Qualifications for Associate Instructors (Teaching Assistants):**
Associate Instructors (Teaching Assistants) must hold BS, MSc or MA and may handle tutorial, laboratory as well as assignment, quiz and problem grading.
<table>
<thead>
<tr>
<th>Syllabus Breakdown:</th>
<th>Course Supervisor is required to distribute syllabus breakdown into 14 units, each unit completed within a week.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedagogical Techniques:</td>
<td>Lecture sessions of each unit (normally, 2-3 lectures) are followed by a discussion session (reinforcing the concepts taught through examples, alternate derivations and proofs) as well as a problem-solving session (teaching skills of problem formulation, qualitative analysis and finding solutions), each of these sessions conducted, separately, by the Associate Instructor at the end of lecture session of every unit. In addition, a review session should be arranged prior to each monthly test (hourly, called because the test is of one-hour duration) and a comprehensive review before the final examination, both sessions conducted by the Course Supervisor.</td>
</tr>
<tr>
<td>Continuous-Evaluation Report:</td>
<td>Course Supervisor (Instructor) is required to prepare a progress report after each hourly mentioning tentative grade (cumulative grade based on all hourlies, quizzes, problem sets and assignments taken to date) and attendance record. It may, also, include qualitative description of student’s weaknesses and areas needing special attention. Laboratory and theory portions are separate passing heads. This report is discussed with the student as well as student’s parents, if unsatisfactory.</td>
</tr>
<tr>
<td>Grading of Problem Sets and Assignments:</td>
<td>An Associate Instructor (A Teaching Assistant) is available to mark problem sets (weekly), quizzes (weekly) and assignments (one or two during the semester).</td>
</tr>
<tr>
<td>Withdrawal Policy:</td>
<td>Student shall have the option to withdraw from a course during a period of 45 days from the start of course. This is possible, only, after the student submits the required forms in the Semester-Examination Section through the Chairman of Department. The first progress report (mentioning tentative grade of the student after the first hourly) is</td>
</tr>
</tbody>
</table>
supposed to be available before the expiry of withdrawal period. A grade of ‘W’ is to be assigned in such a course, and it is not counted in the computation of GPA (Grade-Point Average).

**Final Examinations:** Final Examination (Part I) shall be set up and graded by the Course Supervisor. Final Examination (Part II) shall be set up and graded by a faculty member designated by Board of Studies. The student is supposed to attempt Part I and Part II on separate answer books.

**Marks Breakdown:** Passing grade is set at 50% (65% in foundation and 60% in major courses). Table 1 lists marks breakdowns for courses without laboratory (3 + 0) and courses with laboratory (2 + 1), respectively. Table 2 gives the equivalence of numerical and alphabetical grades.

**Table 1. Marks Breakdown for Courses**

<table>
<thead>
<tr>
<th>Item</th>
<th>Maximum Marks for courses without Laboratory (3 + 0)</th>
<th>Maximum Marks for courses with Laboratory (2 + 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Tests (2-3) a</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>Quizzes (10-14) b</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Problem Sets (10-14) c</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Assignments d</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Laboratory</td>
<td>--</td>
<td>20</td>
</tr>
<tr>
<td>Final Examination</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

a 3 monthly tests (hourlies) for courses without laboratory, best 2 counted; 2 monthly tests (hourlies) for courses with laboratory, best of the 2 counted

b All quizzes are surprise and conducted at the beginning of class/seminar; best 10 counted
c One problem set given each week; best 10 counted
d 2 for courses without laboratory; 1 for courses with laboratory
Table 2. Numerical and Alphabetical Grades

<table>
<thead>
<tr>
<th>Numerical Grade</th>
<th>Alphabetical Grade</th>
<th>Grade Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>90-100</td>
<td>A</td>
<td>4.00</td>
</tr>
<tr>
<td>85-89</td>
<td>A</td>
<td>4.00</td>
</tr>
<tr>
<td>80-84</td>
<td>A</td>
<td>3.67</td>
</tr>
<tr>
<td>75-79</td>
<td>B(^{+})</td>
<td>3.33</td>
</tr>
<tr>
<td>71-74</td>
<td>B</td>
<td>3.00</td>
</tr>
<tr>
<td>68-70</td>
<td>B(^{-})</td>
<td>2.67</td>
</tr>
<tr>
<td>64-67</td>
<td>C(^{+})</td>
<td>2.33</td>
</tr>
<tr>
<td>60-63</td>
<td>C</td>
<td>2.00</td>
</tr>
<tr>
<td>57-59</td>
<td>C(^{-})</td>
<td>1.67</td>
</tr>
<tr>
<td>53-56</td>
<td>D(^{+})</td>
<td>1.33</td>
</tr>
<tr>
<td>50-52</td>
<td>D</td>
<td>1.00</td>
</tr>
<tr>
<td>0-49</td>
<td>F</td>
<td>Zero</td>
</tr>
</tbody>
</table>

Table 3. The BS Scheme of Studies: Structure

<table>
<thead>
<tr>
<th>#</th>
<th>Categories</th>
<th>No. of Courses</th>
<th>Credit Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Compulsory Requirements (no choice)</td>
<td>9</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>General Courses (to be chosen from other departments)</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>Discipline-Specific-Foundation Courses</td>
<td>10</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>Major Courses (including seminars)</td>
<td>12</td>
<td>44</td>
</tr>
<tr>
<td>5</td>
<td>Electives (including project in lieu of free elective)</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td><strong>43</strong></td>
<td><strong>133</strong></td>
</tr>
</tbody>
</table>

Table 4. The BS Scheme of Studies: Layout/Framework

<table>
<thead>
<tr>
<th>Compulsory Requirements</th>
<th>General Courses</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Title</strong></td>
<td><strong>Cr. Hr.</strong></td>
</tr>
<tr>
<td>1. English I (English Structure I)</td>
<td>3</td>
</tr>
<tr>
<td>2. English II (English Structure II)</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Foundation Courses</td>
<td>Major Courses + Seminars</td>
</tr>
<tr>
<td>--------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>Title</td>
<td>Cr. Hr.</td>
</tr>
<tr>
<td>4. English IV (Technical Writing)</td>
<td>3</td>
</tr>
<tr>
<td>5. Introduction to Computers</td>
<td>3</td>
</tr>
<tr>
<td>6. Islamic Studies</td>
<td>2</td>
</tr>
<tr>
<td>7. Mathematics I (Programming Languages for Mathematicians)</td>
<td>4</td>
</tr>
</tbody>
</table>

**Total 25 24**

<table>
<thead>
<tr>
<th>Title</th>
<th>Cr. Hr.</th>
<th>Title</th>
<th>Cr. Hr.</th>
<th>Title</th>
<th>Cr. Hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Algebra I</td>
<td>3</td>
<td>1. Discrete</td>
<td>3</td>
<td><strong>Within the</strong></td>
<td></td>
</tr>
<tr>
<td>2. Algebra II</td>
<td>3</td>
<td>Structures</td>
<td></td>
<td><strong>field of</strong></td>
<td></td>
</tr>
<tr>
<td>3. Algebra III</td>
<td>3</td>
<td>2. Number Theory</td>
<td>3</td>
<td><strong>specialization</strong></td>
<td></td>
</tr>
<tr>
<td>6. Calculus III</td>
<td>3</td>
<td>Analysis</td>
<td>2. Elective-A II</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>7. Complex Analysis</td>
<td>3</td>
<td>Classical</td>
<td>3</td>
<td>3. Elective-A III</td>
<td>3</td>
</tr>
<tr>
<td>8. Ordinary-Diff. Eqns.</td>
<td>3</td>
<td>Mathematical Spaces</td>
<td>3</td>
<td><strong>Outside the</strong></td>
<td></td>
</tr>
<tr>
<td>9. Real Analysis I</td>
<td>2</td>
<td>Mathematical Statistics</td>
<td>3</td>
<td><strong>field of</strong></td>
<td></td>
</tr>
<tr>
<td>10. Real Analysis II</td>
<td>2</td>
<td>Numerical Analysis</td>
<td>4</td>
<td><strong>specialization</strong></td>
<td></td>
</tr>
<tr>
<td>11. Optimization Partial-Diff. Eqns.</td>
<td>3</td>
<td>Analysis</td>
<td>OR Project</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. Modeling &amp; Simulation</td>
<td>3</td>
<td>Optimization</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Theory</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>Seminars</td>
<td>8</td>
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</tbody>
</table>
Table 5. The BS Scheme of Studies:
Semester-Wise Breakdown

<table>
<thead>
<tr>
<th>First Year</th>
<th>Second Year</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FIRST YEAR</strong></td>
<td><strong>Second Semester</strong></td>
</tr>
<tr>
<td>#</td>
<td>Course Title</td>
</tr>
<tr>
<td>1</td>
<td>Calculus I</td>
</tr>
<tr>
<td>2</td>
<td>Discrete Structures</td>
</tr>
<tr>
<td>3</td>
<td>English Structure I</td>
</tr>
<tr>
<td>4</td>
<td>General-A I</td>
</tr>
<tr>
<td>5</td>
<td>General-B I</td>
</tr>
<tr>
<td>6</td>
<td>Guest/Students’ Seminar I</td>
</tr>
<tr>
<td>7</td>
<td>Islamic Studies</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>16 + 2</strong></td>
</tr>
</tbody>
</table>

**Learning Outcomes:** After completing first year, the student should be able to utilize basic tools for computations in order to solve problems of mathematics.

<table>
<thead>
<tr>
<th>SECOND YEAR</th>
<th>Fourth Semester</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SECOND YEAR</strong></td>
<td><strong>Fourth Semester</strong></td>
</tr>
<tr>
<td>#</td>
<td>Course Title</td>
</tr>
<tr>
<td>1</td>
<td>Calculus III</td>
</tr>
<tr>
<td>2</td>
<td>Communication Skills for Mathematicians</td>
</tr>
<tr>
<td>3</td>
<td>General-A III</td>
</tr>
<tr>
<td>4</td>
<td>General-B III</td>
</tr>
<tr>
<td>5</td>
<td>Guest/Students’ Seminar III</td>
</tr>
<tr>
<td>6</td>
<td>Prog. Languages for Mathematicians</td>
</tr>
<tr>
<td>7</td>
<td>English Structure II</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>13 + 4</strong></td>
</tr>
</tbody>
</table>

**Learning Outcomes:** After completing second year, the student should be able to understand proof and to write a formal proof for theoretical applications.
THIRD YEAR

<table>
<thead>
<tr>
<th>#</th>
<th>Course Title</th>
<th>Cr. Hr.</th>
<th>#</th>
<th>Course Title</th>
<th>Cr. Hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Algebra II</td>
<td>3 + 0</td>
<td>1</td>
<td>Algebra III</td>
<td>3 + 0</td>
</tr>
<tr>
<td>2</td>
<td>Vect. &amp; Tensor Anal.</td>
<td>3 + 0</td>
<td>2</td>
<td>Classical Mechanics</td>
<td>3 + 0</td>
</tr>
<tr>
<td>3</td>
<td>Guest/Students’ Seminar V</td>
<td>1 + 0</td>
<td>3</td>
<td>Complex Analysis</td>
<td>3 + 0</td>
</tr>
<tr>
<td>4</td>
<td>Math. Statistics</td>
<td>3 + 0</td>
<td>4</td>
<td>Guest/Students’ Seminar VI</td>
<td>1 + 0</td>
</tr>
<tr>
<td>5</td>
<td>Numerical Analysis</td>
<td>3 + 0</td>
<td>5</td>
<td>Mathematical Spaces</td>
<td>3 + 0</td>
</tr>
<tr>
<td>6</td>
<td>Ordinary-Diff. Eqns.</td>
<td>3 + 0</td>
<td>6</td>
<td>Partial-Diff. Eqns.</td>
<td>3 + 0</td>
</tr>
<tr>
<td>7</td>
<td>Real Analysis I</td>
<td>2 + 0</td>
<td>7</td>
<td>Real Analysis II</td>
<td>2 + 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Total</strong> 18 + 0</td>
<td>7</td>
<td></td>
<td><strong>Total</strong> 18+3</td>
</tr>
</tbody>
</table>

Learning Outcomes: After completing third year, the student should know the main branches of mathematics, i.e., analysis, topology, differential equations, mechanics, numerical analysis, probability and statistics and to apply these theories in practical problems.

FOURTH YEAR

<table>
<thead>
<tr>
<th>#</th>
<th>Course Title</th>
<th>Cr. Hr.</th>
<th>#</th>
<th>Course Title</th>
<th>Cr. Hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Elective-A I</td>
<td>3 + 0</td>
<td>1</td>
<td>Elective-A III</td>
<td>3 + 0</td>
</tr>
<tr>
<td>2</td>
<td>Elective-A II</td>
<td>3 + 0</td>
<td>2</td>
<td>Elective-B OR Project</td>
<td>3 + 0</td>
</tr>
<tr>
<td>3</td>
<td>Functional Analysis</td>
<td>3 + 0</td>
<td>3</td>
<td>Guest/Students’ Seminar VIII</td>
<td>1 + 0</td>
</tr>
<tr>
<td>4</td>
<td>Guest/Students’ Seminar VII</td>
<td>1 + 0</td>
<td>4</td>
<td>Modeling &amp; Simulation</td>
<td>2 + 1</td>
</tr>
<tr>
<td>5</td>
<td>Technical Writing</td>
<td>2 + 1</td>
<td>5</td>
<td>Optimization Theory</td>
<td>3 + 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Total</strong> 12 + 1</td>
<td></td>
<td></td>
<td><strong>Total</strong> 12 + 1</td>
</tr>
</tbody>
</table>

Learning Outcomes: After completing fourth year, the student should choose the field of specialization and write research paper.
The BS Course Contents

ALGEBRA I
Prerequisite(s): Mathematics at intermediate level
Credit Hours: 3 + 0

Specific Objectives of the Course:
This is the first course in groups, matrices and linear algebra, which provides basic background needed for all mathematics majors, a prerequisite for many courses. Many concepts presented in the course are based on the familiar setting of plane and real three-space, and are developed with an awareness of how linear algebra is applied.

Course Outline:
Group Theory: Basic axioms of a group with examples, abelian groups, center of a group, derived subgroup of a group, subgroups generated by subset of a group, system of generators, cyclic groups, cosets and quotient sets, Lagrange’s theorem, introduction to permutations, even and odd permutations, cycles, lengths of cycles, transpositions, symmetric group, alternating groups, rings, finite and infinite fields (definition and examples), vector spaces, subspaces, linear span of a subset of a vector space, bases and dimensions of a vector space
Algebra of Matrices: Determinants, matrix of a linear transformation. row and column operations, rank, inverse of matrices, group of matrices and subgroups, orthogonal transformation, eigenvalue problem with physical significance

Recommended Books:
Herstein IN, Topics in Algebra (2nd edition), John Wiley, New York
Leon SJ, Linear Algebra with Applications (6th edition), 2002, Prentice Hall,
Englewood Cliffs, NJ, USA
Nicholson WK, Elementary Linear Algebra with Applications (2nd edition),...
ALGEBRA II
Prerequisite(s): Algebra I
Credit Hours: 3 + 0

Specific Objectives of the Course:
This is a course in advanced abstract algebra, which builds on the concepts learnt in Algebra I.

Course Outline:
Group Theory: Normalizers and centralizers of a subset of a group, congruency classes of a group, normal subgroup, quotient groups, conjugacy relation between elements and subgroups, homomorphism and isomorphism between groups, Homomorphism and isomorphism theorems, group of automorphisms, finite p-groups, internal and external direct products, group action on sets, isotropy subgroups, orbits, 1st, 2nd and 3rd Sylow theorems.

Ring Theory: Types of rings, matrix rings, rings of endomorphisms, polynomial rings, integral domain, characteristic of a ring, ideal, types of ideals, quotient rings, homomorphism of rings, fundamental theorem of homomorphism of rings.

Recommended Books:
Allenby RBJT, Rings, Fields and Groups: An Introduction to Abstract Algebra, 1983, Edward Arnold
Farleigh JB, A First Course in Abstract Algebra (7th edition), Addison-Wesley, Reading, Ma., USA
Macdonald ID, The Theory of Groups, 1975, Oxford Clarendon Press, Ma., USA

ALGEBRA III
Prerequisite(s): Algebra II
Credit Hours: 3 + 0

Specific Objectives of the Course:
This is a course in abstract linear algebra. The majority of follow up courses in both pure and applied mathematics assume the material covered in this course.

Course Outline:
Vector spaces; sums and direct sums of subspaces of a finite dimensional vector space, Dimension theorem, linear transformation, null space, image space of linear transformation, rank and nullity of a linear transformation, relation between rank, nullity and dimension of the domain of a linear transformation, matrix of linear transformation, change of basis, inner product spaces, projection of a vector along another vector, norm of a vector, Cauchy Schwartz inequality, Orthogonal and orthonormal basis, similar matrices and diagonalization of a matrix, Home (V,W), dimension and basis of Home (V.W), dual space and dual basis, annihilators.

Recommended Books:

CALCULUS I
Prerequisite(s): Mathematics at intermediate level
Credit Hours: 3 + 0

Specific Objectives of the Course:
This is the first course of the basic sequence, Calculus I-III, serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes basic concepts and skills needed for mathematical manipulation. Calculus I & II focus on the study of functions of a single variable.
Course Outline:
Limits and continuity; derivative of a function and its applications; optimization problems; mean value theorem (Taylor’s theorem and the infinite Taylor series with applications) and curve sketching; anti-derivative and integral; definite integral and applications; the fundamental theorem of calculus; inverse functions (Chapters 1-6 of the text)

Recommended Books:
Thomas GB, Finney AR, Calculus (11th edition), 2005, Addison-Wesley, Reading, Ma, USA
CALCULUS II
Prerequisite(s): Calculus I
Credit Hours: 3 + 0

Specific Objectives of the Course:
This is the second course of the basic sequence Calculus I-III serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes basic concepts and skills needed for mathematical manipulation. As continuation of Calculus I, it focuses on the study of functions of a single variable.

Course Outline:
Continuation of Calculus I: Techniques of integration; further applications of integration; parametric equations and polar coordinates; sequences and series; power series representation of functions (Chapters 7-10 of the text)

Recommended Books:
Thomas GB, Finney AR, Calculus (11th edition), 2005, Addison-Wesley, Reading, Ma, USA

CALCULUS III
Prerequisite(s): Calculus II
Credit Hours: 3 + 0

Specific Objectives of the Course:
This is the third course of the basic sequence Calculus I-III serving as the foundation of advanced subjects in all areas of mathematics.

Course Outline:
This course covers vectors and analytic geometry of 2 and 3 dimensional spaces; vector-valued functions and space curves; functions of several variables; limits and continuity; partial derivatives; the chain rule; double and triple integrals with applications; line integrals; the Green theorem; surface area and surface integrals; the Green, the divergence and the Stokes theorems with applications (Chapters 11-14 of the text)
Recommended Books:
Thomas GB, Finney AR, *Calculus* (11th edition), 2005, Addison-Wesley, Reading, Ma, USA

CLASSICAL MECHANICS
Prerequisite(s): Vector and Tensor Analysis
Credit Hours: 3 + 0

Specific Objectives of the Course:
This course builds grounding in principles of classical mechanics, which are to be used while studying quantum mechanics, statistical mechanics, electromagnetism, fluid dynamics, space-flight dynamics, astrodynamics and continuum mechanics.

Course Outline:
Particle kinematics, radial and transverse components of velocity and acceleration, circular motion, motion with a uniform acceleration, the Newton laws of motion (the inertial law, the force law and the reaction law), newtonian mechanics, the newtonian model of gravitation, simple-harmonic motion, damped oscillations, conservative and dissipative systems, driven oscillations, nonlinear oscillations, calculus of variations, Hamilton's principle, lagrangian and hamiltonian dynamics, symmetry and conservation laws, Noether's theorem, central-force motion, two-body problem, orbit theory, Kepler's laws of motion (the law of ellipses, the law of equal areas, the harmonic law), satellite motion, geostationary and polar satellites, kinematics of two-particle collisions, motion in non-inertial reference frame, rigid-body dynamics (3-D-rigid bodies and mechanical equivalence, motion of a rigid body, inverted pendulum and stability, gyroscope)

Recommended Books:
Goldstein H, *Classical Mechanics* (2nd edition), 1980, Addison-Wesley, Reading, Ma, USA
COMPLEX ANALYSIS

Prerequisite(s): Real Analysis I
Credit Hours: 3 + 0

Specific Objectives of the Course:
This is an introductory course in complex analysis, giving the basics of the theory along with applications, with an emphasis on applications of complex analysis and especially conformal mappings. Students should have a background in real analysis (as in the course Real Analysis I), including the ability to write a simple proof in an analysis context.

Course Outline:
The algebra and the geometry of complex numbers, Cauchy-Riemann equations, harmonic functions, elementary functions, branches of the logarithm, complex exponents. Contours and contour integrals, the Cauchy-Goursat Theorem, Cauchy integral formulas, the Morera Theorem, maximum modulus principle, the Liouville theorem, fundamental theorem of algebra. Convergence of sequences and series, the Taylor series, the Laurent series, uniqueness of representation, zeros of analytic functions. Residues and poles and the residue theorem, evaluation of improper integrals involving trigonometric functions, integrals around a branch point, the argument principle, the Roche theorem.

Recommended Text:

COMPUTING TOOLS FOR MATHEMATICIANS

Prerequisite(s): Programming Languages for Mathematicians
Credit Hours: 1 + 1

Specific Objectives of the Course:
The purpose of this course is to teach students the use of mathematical software like MATLAB, MAPLE, MATHEMATICA for solving computationally-difficult problems in mathematics. The student shall become well versed in using at least one mathematical software and shall learn a number of techniques that are useful in calculus as well as in other areas of mathematics.
Course Outline:
The contents of the course are not fixed, however the following points should be kept in mind while teaching the course. The course should be taught in a computer lab setting. Besides learning to use the software, the students must be able to utilize the software to solve computationally difficult problems in calculus and other areas of mathematics. At the end of the course, the students should have a good command on at least two of the three programs mentioned above.

Recommended Books:

DISCRETE STRUCTURES
Prerequisite(s): Mathematics at intermediate level
Credit Hours: 3 + 0

Specific Objectives of the Course:
This course shall assume background in number theory. It lays a strong emphasis on understanding and utilizing various strategies for composing mathematical proofs.

Course Outline:
*Set and Relations*: Basic notions, set operations, Venn diagrams, extended-set operations, indexed family of sets, countable and uncountable sets, relations, cardinality, equivalence relations, congruence, partitions, partial order, representation of relations, mathematical induction.
*Elementary Logic*: Logics of order zero and one, Propositions and connectives, truth tables, conditionals and biconditionals, quantifiers, methods of proof, proofs involving quantifiers.

Recommended Text:
FUNCTIONAL ANALYSIS

Prerequisite(s): Complex Analysis
Credit Hours: 3 + 0

Specific Objectives of the Course:
This course extends methods of linear algebra and analysis to spaces of functions, in which the interaction between algebra and analysis allows powerful methods to be developed. The course will be mathematically sophisticated and will use ideas both from linear algebra and analysis.

Course Outline:

Metric Spaces: A quick review, completeness and convergence, completion.
Banach-Fixed-Point Theorem: Applications in Differential and Integral equations
Inner-Product Spaces: Inner-product space, Hilbert space, orthogonal and orthonormal sets, orthogonal complements, Gram-Schmidt orthogonalization process, representation of functionals, Reiz-representation theorem, weak and weak* Convergence.

Recommended Books:
Friedman A, Foundations of Modern Analysis, 1982, Dover

MATHEMATICAL SPACES
Prerequisite(s): Discrete Structures, Real Analysis I
Credit Hours: 2 + 0

Specific Objectives of the Course:
This course is designed primarily to develop pure mathematical skills of
students. Students will need some background in writing proofs. They
will lean notions of spaces, metric, measure and topology

**Course Outline:**

*Notion of Spaces:* Example of set, group, field, ring, affine space,
Banach space, normed space, Hilbert space (Simmon)

  a) *Notion of Topology:* Calculus on manifolds, continuity of
    functions on spaces, neighborhoods, topological spaces, finer
    and weaker topologies, homomorphism, homomorphic spaces,
    compactness, connectedness, normal spaces, Urysohn’s lemma
    (Munkres)
  
  b) *Notion of Metric:* Metric space, complete metric space, Baire
     category theorem, metrization of spaces (Friedmann)
  
  c) *Notion of Measure:* Spaces with measure, measurable function,
     idea of $\sigma$ – fields (Holmos)

**Recommended Books:**

NJ, USA
Simmon GF, *Introduction to Topology and Modern Analysis*, 1963,
McGraw Hill, New York

**MATHEMATICAL STATISTICS**

**Prerequisite(s):** Probability Theory

**Credit Hours:** 3 + 0

**Specific Objectives of the Course:**

In the course “Probability Theory” the students learnt how to set up
mathematical models of processes and systems that are affected by
*chance*. In the present course the students would learn how to check
these models against reality, to determine whether they are
reliable/accurate enough for practical purposes or otherwise. This helps
in making predictions and decisions

**Course Outline:**

Sampling theory: sampling distributions; sampling proce-dures;
estimation of parameters: estimation of mean, variance; confidence
intervals; decision theory: hypothesis testing and decision making; types of errors in tests; quality control; control charts for mean, standard deviation, variance, range; goodness of fit, chi-square test. Regression analysis; method of least squares; correlation analysis

**Recommended Books:**

**MODELING AND SIMULATION**

**Prerequisite(s):** Partial-Differential Equations

**Credit Hours:** 2 + 1

**Specific Objectives of the Course:**
Mathematics is used in many areas such as engineering, ecological systems, biological systems, financial systems, economics, etc. In all such applications one approximates the actual situation by an idealized model. This is an introductory course of modeling, consisting of three parts: modeling with ordinary differential equations and their systems; partial differential equations; and integral equations. The course will not be concerned with the techniques for solving the equations but with setting up the equations in specific applications. Whereas the first two types of equations have already been dealt with, the third type has not. Consequently, solutions of the former will be discussed but of the latter will barely be touched upon.

**Course Outline:**
Concepts of model, modeling and simulation, functions, linear equations, linear-differential equations, nonlinear-differential equations and integral equations as models, introduction to simulation techniques

*Ordinary-Differential Equations:* Modeling with first order differential equations: Newton’s law of cooling; radioactive decay; motion in a gravitational field; population growth; mixing problem; Newtonian mechanics. Modeling with second order differential equations: vibrations; application to biological systems; modeling with periodic or impulse
forcing functions. Modeling with systems of first order differential equations; competitive hunter model; predator-prey model.

Partial-Differential Equations: Methodology of mathematical modeling; objective, background, approximation and idealization, model validation, compounding. Modeling wave phenomena (wave equation); shallow water waves, uniform transmission line, traffic flow, RC circuits. Modeling the heat equation and some application to heat conduction problems in rods, lamina, cylinders etc. Modeling the potential equation (Laplace equation), applications in fluid mechanics, gravitational problems. Equation of continuity.

Simulation: Techniques of simulation (students are required to simulate at least one system)

Recommended Books:
Giordano FR, Weir MD, Differential Equations: A Modeling Approach, 1994, Addison-Wesley, Reading, Ma, USA (suggested text)
Myint UT, Debnath L, Partial Differential Equations for Scientists and Engineers (3rd edition), 1987, North Holland, Amsterdam

NUMBER THEORY
Prerequisite(s): Calculus I, Discrete Structures
Credit Hours: 3 + 0

Specific Objectives of the Course:
This course shall assume no experience or background in number theory or theoretical mathematics. The course introduces various strategies for composing mathematical proofs.

Course Outline:
Divisibility, euclidean algorithm, GCD and LCM of 2 integers, properties of prime numbers, fundamental theorem of arithmetic (UFT), congruence relation, residue system, Euler’s phi-function, solution of system of linear congruences, congruences of higher degree, Chinese remainder theorem, Fermat’s little theorem, Wilson’s theorem and applications, primitive roots and indices; integers belonging to a given exponent (mod p), primitive roots of prime and composite moduli, indices, solutions of congruences using indices., quadratic residues, composite moduli,
quadratic residues of primes, the Legendre symbol, the Quadratic
reciprocity law, the Jacobi symbol, Diophantine equations

**Recommended Books:**
Grosswald E, *Topics from the Theory of Numbers*, The Macmillan
Company
LeVeque WJ, *Topics in Number Theory*, Vol.1, Addison-Wesley,
Reading, Ma, USA
Niven I, Zuckerman HS, *An Introduction to The Theory of Numbers*,
Wiley Eastern
2000,
Addison-Wesley, Reading, Ma, USA (suggested text)

**NUMERICAL ANALYSIS**

**Prerequisite(s):** Computing Tools for Mathematicians

**Credit Hours:** 3 + 0

**Specific Objectives of the Course:**
This course is designed to teach the students about numerical methods
and their theoretical bases. The students are expected to know computer
programming to be able to write program for each numerical method.
Knowledge of calculus and linear algebra would help in learning these
methods.

**Course Outline:**
Computer arithmetic, approximations and errors; methods for the
solution of nonlinear equations and their convergence: bisection method,
regula falsi method, fixed point iteration method, Newton-Raphson
method, secant method; error analysis for iterative methods.
Interpolation and polynomial approximation: Lagrange interpolation,
Newton’s divided difference, forward-difference and backward-
difference formulae, Hermite interpolation. Numerical integration and
error estimates: rectangular rule, trapezoidal rule, Simpson’s one-three
and three-eight rules. Numerical solution of systems of algebraic linear
equations: Gauss-elimination method, Gauss-Jordan method; matrix
inversion; LU-factorization; Doolittle’s, Crout’s, Cholesky’s methods;
Gauss-Seidel and Jacobi methods
Recommended Books:

**OPTIMIZATION THEORY**

**Prerequisite(s):** Algebra I, Real Analysis I

**Credit Hours:** 3 + 0

**Specific Objectives of the Course:**
The main objective is to teach the basic notions and results of mathematical programming and optimization. The focus will be to understand the concept of optimality conditions and the construction of solutions. Students should have a good background in analysis, linear algebra and differential equations.

**Course Outline:**
Linear programming: simplex method, duality theory, dual and primal-dual simplex methods. Unconstrained optimization: optimality conditions, one-dimensional problems, multi-dimensional problems and the method of steepest descent. Constrained optimization with equality constraints: optimality conditions, Lagrange multipliers, Hessians and bordered Hessians. Inequality constraints and the Kuhn-Tucker Theorem. The calculus of variations, the Euler-Lagrange equations, functionals depending on several variables, variational problems in parametric form, transportation models and networks

**Recommended Books:**
Luenberger DG, *Introduction to Linear and Non-Linear Programming*, 1973, Addison-Wesley, Reading, Ma, USA

**ORDINARY-DIFFERENTIAL EQUATIONS**
Prerequisite(s): Calculus III, Computing Tools for Mathematicians
Credit Hours: 3 + 0

Specific Objectives of the Course:
This course provides the foundation of all advanced subjects in Mathematics. Strong foundation and applications of Ordinary Differential Equations is the goal of the course.

Course Outline:
Introduction; formation, solution and applications of first-order-differential equations; formation and solution of higher-order-linear-differential equations; differential equations with variable coefficients; Sturm-Liouville (S-L) system and boundary-value problems; series solution and its limitations; the Frobenius method, solution of the Bessel, the hypergeometric, the Legendre and the Hermite equations, properties of the Bessel function

Recommended Text:

PARTIAL-DIFFERENTIAL EQUATIONS
Prerequisite(s): Real Analysis I, Ordinary-Differential Equations
Credit Hours: 3 + 0

Specific Objectives of the Course:
The course provides a foundation to solve Partial Differential Equations with special emphasis on wave, heat and Laplace equations. Formulation and some theory of these equations are also intended.

Course Outline:
First-order-partial-differential equations; classification of second-order PDE; canonical form for second-order equations; wave, heat and the Laplace equation in Cartesian, cylindrical and spherical-polar coördinates; solution of partial differential equation by the methods of: separation of variables; the Fourier, the Laplace and the Hankel transforms, non-homogeneous-partial-differential equations

Recommended Text:
PROBABILITY THEORY
Prerequisite(s): Calculus III
Credit Hours: 3 + 0

Specific Objectives of the Course:
This course is designed to teach the students how to handle data numerically and graphically. If data are influenced by chance effect, the concepts and rules of probability theory may be employed, being the theoretical counterpart of the observable reality, whenever chance is at work.

Course Outline:
Introduction to probability theory; random variables; probability distributions; mean, standard deviation, variance and expectation. Binomial, negative binomial, Poisson, geometric, hypergeometric and normal distributions; normal approximation to binomial distribution; distributions of 2 random variables.

Recommended Books:
Sincich T, Statistics by Examples, 1990, Dellen Publishing Company

PROGRAMMING LANGUAGES FOR MATHEMATICIANS
Prerequisite(s): Calculus II
Credit Hours: 3 + 1

Specific Objectives of the Course:
The purpose of this course is to introduce students to operating systems and environments

Course Outline:
Introduction to operating systems, one Language (FORTRAN or C/C++), building blocks, variables, input/output, loops (FOR, WHILE, DO), decisions (IF, IF ELSE, ELSE IF) construct switch statement, conditional statement, function that returns a value using argument to pass data to another function, external variable, arrays and strings, pointers, structure, files and introduction to object-oriented programming
Recommended Books:

REAL ANALYSIS I
Prerequisite(s): Calculus III
Credit Hours: 2 + 0

Specific Objectives of the Course:
This is the first rigorous course in analysis and has a theoretical emphasis. It rigorously develops the fundamental ideas of calculus and is aimed to develop the students’ ability to deal with abstract mathematics and mathematical proofs.

Course Outline:
Ordered sets, supremum and infimum, completeness properties of the real numbers, limits of numerical sequences; limits and continuity, properties of continuous functions on closed bounded intervals; derivatives in one variable; the mean value theorem; Sequences of functions, power series, point-wise and uniform convergence. Functions of several variables: open and closed sets and convergence of sequences in \( \mathbb{R}^n \); limits and continuity in several variables, properties of continuous functions on compact sets; differentiation in \( n \)-space; the Taylor series in \( \mathbb{R}^n \) with applications; the inverse and implicit function theorems.

Recommended Books:
Brabenec RL, Introduction to Real Analysis, 1997, PWS Publishing Company

REAL ANALYSIS II
Prerequisite(s): Real Analysis I
Credit Hours: 2 + 0
Specific Objectives of the Course:
A continuation of Real Analysis I, this course rigorously develops integration theory. Like Real Analysis I, Real Analysis II emphasizes proofs.

Course Outline:

Recommended Books:

VECTOR AND TENSOR ANALYSIS
Prerequisite(s): Calculus II
Credit Hours: 3 + 0

Specific Objectives of the Course:
This course shall assume background in calculus. It covers basic principles of vector analysis, which are used in mechanics

Course Outline:
3-D vectors, summation convention, kronecker delta, Levi-Civita symbol, vectors as quantities transforming under rotations with $\varepsilon_{ijk}$ notation, scalar- and vector-triple products, scalar- and vector-point functions, differentiation and integration of vectors, line integrals, path independence, surface integrals, volume integrals, gradient, divergence and curl with physical significance and applications, vector identities, Green's theorem in a plane, divergence theorem, Stokes' theorem, coordinate systems and their bases, the spherical-polar- and the cylindrical-coordinate meshes, tensors of first, second and higher orders,
algebra of tensors, contraction of tensor, quotient theorem, symmetric and skew-symmetric tensors, invariance property, application of tensors in modeling anisotropic systems, study of physical tensors (moment of inertia, index of refraction, etc.), diagonalization of inertia tensor as aligning coordinate frame with natural symmetries of the system.

Recommended Books:

The BS Electives

<table>
<thead>
<tr>
<th>Course Name</th>
<th>Course Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract Algebra I</td>
<td>Abstract Algebra II</td>
</tr>
<tr>
<td>Advanced Calculus I</td>
<td>Advanced Calculus I</td>
</tr>
<tr>
<td>(Numerical Solutions of PDE)</td>
<td>(Integral Equations)</td>
</tr>
<tr>
<td>Advanced Numerical Analysis I</td>
<td>Advanced Numerical Analysis II</td>
</tr>
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<td>Astronomy I</td>
<td>Astronomy II</td>
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<td>Differential Geometry I</td>
<td>Differential Geometry II</td>
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<td>Electromagnetism I</td>
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<td>Fluid Dynamics I</td>
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<td>Group Theory I</td>
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<td>(Fundamentals)</td>
<td>(Study of Symmetries)</td>
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<td>History and Philosophy of Mathematics I</td>
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<td>Measure Theory I</td>
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<td>Modern Algebra I</td>
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<td>(Galios Theory &amp; Applications)</td>
<td>(Commutative Rings &amp; Fields)</td>
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<td>Nonlinear Systems I</td>
<td>Nonlinear Systems II</td>
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<tr>
<td>Numerical Solutions of Partial-Differential Equations I</td>
<td>Numerical Solutions of Partial-Differential Equations II</td>
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<tr>
<td>Operations Research I</td>
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<tr>
<td>Projective Geometry I</td>
<td>Projective Geometry II</td>
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<tr>
<td>Quantum Mechanics I</td>
<td>Quantum Mechanics II</td>
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<td>Relativity I</td>
<td>Relativity II</td>
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<td>Software Engineering I</td>
<td>Software Engineering II</td>
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<td>(Design &amp; Development)</td>
<td>(Analysis)</td>
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<td>Theory of Processes I</td>
<td>Theory of Processes II (Renewal)</td>
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(Stochastic Processes) Processes & Theory of Ques
Theory of Splines I Theory of Splines II
Topology I Topology II
(Topological-Dimension Theory) (Differential Topology)
RECOMMENDATIONS
(To make the curriculum-development effort effective)

National-Curriculum-Revision Committee in Mathematics, unanimously, made the following recommendations:

i. Every public/private sector university should have a mathematics department employing full-time faculty.

ii. HEC should pay for the membership of at least one international professional society membership and subscription of at least two international journals for each active faculty member.

iii. National conferences in mathematics should have a session devoted to oral and poster presentations of BS students.

iv. National conferences in mathematics should have a session devoted to pedagogical skills in mathematics.

v. BS students presenting paper in international conference should receive monetary incentives.

vi. MS/PhD students publishing paper in an international journal should receive monetary incentives.

vii. Every oral and poster presentation as well as journal publication in mathematics by multiple authors must, clearly, describe the nature and the type of contribution made by each author. The presentation or publication should, also, mention the names of study supervisor and study advisor(s), wherever applicable.

viii. Course load for Professor, Associate Professor, Assistant Professor and Lecturer should be 3-credit hours, 6-credit hours 9-credit hours and 12-credit hours, respectively.

ix. For selection/promotion, teaching experience should be counted after 16 years of schooling.

x. Existing faculty members should be selected/promoted to higher grades according to existing criteria. New faculty members may be inducted according to revised criteria.

xi. There should be equal salaries for Mathematics and IT teachers.

xii. Mathematics-research-impact factor should be evaluated separately from other disciplines.

xiii. Top 3 position holders in BS (Mathematics) of public-sector universities should be awarded scholarships to pursue higher education in the country or abroad.

xiv. There should be a provision of teachers-training program (philosophy + contents + pedagogical techniques) to prepare manpower for BS, MS and PhD programs proposed in this document.

xv. In order to make these programs successful, faculty satisfaction is essential. Hence, a committee should be formed to formulate career-development protocols for the faculty.

xvi. Textbook writing by Pakistani experts to be encouraged, monetary
incentives and sabbatical leave to bring out quality publications; young faculty members should be compiling their lecture notes as e-books, which could come out regular hard-copy books after they have been teaching the course for 5 consecutive years.

xvii Evaluation of thesis should be by experts in the field, who may reside in or outside Pakistan. The competence of expert is not to be determined by the expert country of residence, but the number of publications and their impact factor in the area during the last 5 years.

xviii Examination systems must also be modernized in order to maintain standards. A separate forum should chart out examination-conduct procedures, pre- and post-examination exercises to conduct the examinations smoothly, with efficiency and effectiveness.

xix Four-Year BS Program in Mathematics should, also, be implemented in Engineering Universities.

xx Effectiveness of the Four-Year BS Program implemented in various universities should be evaluated.